Study Session 2

Quantitative Methods: Basic Concepts

Candidate Note: This is a lengthy Study Session that, along with Study Session 3, you should plan on reviewing more than once, particularly if any or all of the topic areas are new to you. We suggest reviewing it with a CFA Institute-approved financial calculator so that you can work through the examples and become familiar with the calculations. Study time spent on this Study Session could benefit you greatly on exam day.

The numerical examples shown in the Quantitative Methods Study Sessions are displayed with rounded numbers, but the calculations are done with greater precision. You may find a small amount of what appears to be rounding error, but this is because numbers in intermediate steps are displayed with some rounding. However, exact figures are used throughout the calculations, and the final answers reflect this greater precision.

Note that any necessary key formulas and appendices are located at the end of Study Session 3.

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1. Time Value of Money

Learning Objectives

This summary includes a review and an analysis of the principles set forth by CFA Institute. Upon review of this summary, you should be able to:

Candidate Note: Of all the concepts used in finance, none is more important than the *time value of money*, or *discounted cash flow (DCF) analysis*. It is *vitally* important that you thoroughly understand the material in this summary before advancing to other finance-related subject matter.

Learning Objective: Understand interest rates as required rates of return, discount rates, or opportunity costs.

Interest rates can be viewed a number of ways. For example, if you are a creditor lending money, the interest rate you charge the borrower is your **required rate of return**. It *is the minimum rate or return that you are willing to accept in order to let someone borrow your money*.

Another interpretation is that an interest rate can be thought of as a discount rate. Suppose you win the lottery, with a jackpot of $20 million, and you are given a choice of receiving $500,000 per year for 40 years, starting immediately, or a lump sum (before tax) of $5,378,477.90. In this case, the lottery officials used a 10% rate of interest to compute the present value of the 40 half-million dollar annual payments. The **discount rate** *is the interest rate used to compute present value*, i.e., what the stream of payments is worth in today's dollars.

Finally, an interest rate can be thought of as an opportunity cost. Suppose you are the CFO of a major bank, and you are deciding how to deploy $100 million. You could invest it internally or make an acquisition. You believe each alternative to be equally risky, and that deploying the funds internally will provide a return of 15%. Thus, to consider an acquisition instead, the return must equal or exceed this 15% interest rate - called the opportunity cost rate. The **opportunity cost rate** is *the rate of return on the best available investment of equal (or sufficiently similar) risk*. It is an opportunity cost because to make the acquisition you would have to give up the opportunity to invest the $100 million at 15%.

Learning Objective: Understand a nominal interest rate as the sum of a real risk-free rate, expected inflation, and premiums that compensate investors for distinct types of risk.

In a risk-free world, the interest rate is called the risk-free rate. Those are the instances where the cash flows of lending/borrowing contracts are virtually certain. Some developed countries have short-term debt that can be considered risk-free. U.S. Treasury bills (Treasury securities with maturities less than one year), for example, are usually considered risk-free because they are backed by the full faith and credit of the U.S. government. So interest rate on Treasury bills (T-bills) is an example of a risk-free rate of return.

Two factors complicate interest rates in the real world: inflation and risks.

1. **Inflation**. Consumer prices tend to increase over time, so lenders charge not only an *opportunity cost* for postponing consumption but also an *inflation premium* that takes into account the expected increase in consumer prices. The *nominal interest rate* therefore consists of a pure rate of interest, called the *real rate*, and an inflation premium. The inflation premium compensates investors for *expected* changes in price levels and is

based on the next period's *expected* inflation rather than on the actual inflation rate for current or past periods.

2. **Risks**. In contrast to stable governments with the full power to tax to meet their debt obligations, private corporations exhibit varying degrees of uncertainty concerning their ability to repay lenders, as well as posing other liquidity and maturity concerns. The interest rates that lenders charge needs to incorporate various risk premiums. The return that borrowers pay thus comprises the nominal risk-free rate (the pure rate plus an inflation premium) plus various risk premiums, such as default risk premium, liquidity premium, and maturity risk premium.

We can use the following formula to summarize the components of an interest rate:

Nominal interest rate = Nominal risk-free rate + Risk premiums

Where:

Nominal risk-free rate = Real risk-free rate + Expected inflation premium

Which is an approximation to the more precise relationship:

(1 + Nominal risk-free rate) = (1 + Real risk-free rate)(1 + *Expected* inflation)

Also note:

Risk premiums = Default risk premium + Liquidity premium + Maturity risk premium

Thus, an investors' required rate of return (RRR) should follow this relationship:

RRR = Opportunity cost (Time value) of money + Expected inflation premium + Risk premiums

Some Important Points on Inflation

High or variable inflation rates send unreliable price signals to investors (and all economic agents), which are hard to interpret, and often lead to sub-optimal choices, and thus result in less efficient resource allocation across the economy as a whole.

Although the investment decisions are made incorporating the *expected* (anticipated) inflation, the future *unanticipated* inflation is what mostly impacts the returns and actual outcomes of those decisions. Unanticipated inflation changes the results of otherwise sound long-term projects since it engenders greater uncertainty and harder risk measurement and control. Investors and other agents become less willing to make longer-term bets, thus forgoing (or delaying) some advantageous investment projects, which on aggregate may decrease the efficiency of the market.

Unanticipated inflation may be caused by outside shocks, economic policy changes, or interaction of a number of random and unpredictable factors. Unanticipated inflation can be measured ex-post - it is the difference between the actual current inflation for a given period, and the inflation expectation for that period as of the previous period.

Future Value

Learning Objective: Calculate the future value (FV) of a single sum of money.

A dollar in hand is worth more than a dollar to be received in the future because if you had $1 now, you could invest it, earn interest, and end up with more than a dollar in the future. *Today's value of money* is called the **present value** (**PV**) while *the value of money in the future* is called **future value** (**FV**). The process of going from PVs to FVs is called **compounding**, and is defined as *the arithmetic process of earning interest on interest for an investment over a period of time*.

To illustrate, suppose we deposit $1,000 in a bank that pays 6% interest per year. How much would we have after one year?

To begin, we define the following terms:

PV = present value, or the beginning amount. In this example, it is $1,000.

i = interest rate that the bank pays per year. The interest earned is based on the balance at the beginning of each year and is assumed to be paid at the end of the year. In this case i = 6% or 0.06.

INT = amount of interest earned, in dollar terms, during the year. In this case it is 0.06 × $1,000 = $60.

FV_n = future value or ending amount after n years.

n = number of periods involved. In this case $n = 1$.

Therefore:

$$\begin{aligned} FV_n = FV_1 &= PV + INT \\ &= PV + PV(i) \\ &= PV(1 + i) \\ &= \$1{,}000\ (1 + 0.06) = \$1{,}060 \end{aligned}$$

Thus, the **future value** is *the amount to which a cash flow or series of cash flows will grow over a given period of time when compounded at a specified interest rate*.

Suppose we did not withdraw the money after one year and kept the $1,060 in the bank for another four years with the interest rate remaining at 6%. How much money will we have at the end of the fifth year?

Observe the time line:

Year	0	1	2	3	4	5
	|-----------|	-----------|	----------|	----------|	-----------|	
Initial deposit	-1,000	FV_1	FV_2	FV_3	FV_4	FV_5
Interest earned		60.00	63.60	67.42	71.46	75.75

Amount at the end of each period:

FV_n		1,060	1,123.60	1,191.02	1,262.48	1,338.23

At the end of the fifth year, the balance would be $1,338.23. The following explains these calculations:

1. Begin with $1,000 deposited, which is an outflow.
2. Earn $1,000(0.06) = $60 of interest at the end of the first year ($t = 1$) so the total amount at that time is $1,060.
3. Begin year two by reinvesting $1,060 for one year at 6%, earning $1,060(0.06) = $63.60. At the end of the second year you now have $1,060 + $63.60 = $1,123.60.
4. This process continues each year, and because the beginning balance is higher in each succeeding year, the annual interest that is earned increases.
5. At the end of five years, the total interest earned is $338.23 and is reflected in the final balance of $1,338.23.

Numerical Solutions

Mathematically, note that at the end of year two, $1,123.60 is equal to:

$$FV_2 = FV_1(1 + i)\text{, but } FV_1 \text{ is equal to } PV(1 + i)$$

Replace FV_1 with $PV_1(1 + i)$:

$$FV_2 = PV(1 + i)(1 + i) = PV(1 + i)^2 = \$1,000(1 + 0.06)^2 = \$1,123.60$$

Continuing, the balance at the end of year three is:

$$FV_3 = FV_2(1 + i) = PV(1 + i)^3 = \$1,000(1.06)^3 = \$1,191.02$$

Finally, for the fifth year, we have:

$$FV_5 = PV(1 + i)^5 = \$1,000(1.06)^5 = \$1,338.23$$

In general, the formula for the future value of an initial lump sum at the end of *n* years can be found by using:

$$FV_n = PV(1 + i)^n$$

This shows the compounding process at work. The initial deposit gains interest after the first year. This amount (initial deposit plus interest) gains interest at the end of the second year and so on. At the end of the fifth year, the initial deposit has increased to $1,338.23 as a result of the compounding process.

Financial Calculator Solutions (Texas Instruments BA II Plus)

Use these keys:

N = number of periods

I/Y = interest rate per year, in whole percents (enter 6 for 6%)

P/Y = payment periods per year; in the example above, P/Y = 1. For mortgage calculations, P/Y = 12.

PV = present value

PMT = payment. This is used only if the cash flows involve a series of equal or constant payments. If there are no periodic payments, then PMT = 0.

FV = future value

CPT = compute

Key in the appropriate numbers for the N, I, and PV and then press CPT FV to get the future value.

Present Value

Learning Objective: Calculate the present value (PV) of a single sum of money.

In our previous example on FV, $1,000 was invested at 6% for five years and was worth $1,338.23 at the end of the fifth year. It can be shown that you should be indifferent to the choice between $1,000 today and $1,338.23 at the end of five years. The $1,000 is defined as the present value of $1,338.23 due in five years when the opportunity cost rate is 6%. The

<u>opportunity cost rate</u> is *<u>the rate of return on the best available investment of equal (or sufficiently similar) risk</u>*.

Suppose you had the opportunity to buy a security that would be worth $1,338.23 after five years. Should you buy this security? If the price of the security were less than $1,000, you should buy it, because its price would then be less than the $1,000 you would have to spend on a similar-risk alternative to end up with $1,338.23 after five years. Conversely, if the security cost more than $1,000, you should not buy it, because you would have to invest only $1,000 in a similar-risk alternative to end up with $1,338.23 after five years. If the price were exactly $1,000, you should be indifferent - you could either buy the security or not buy it. Therefore, $1,000 is the security's **<u>fair (equilibrium) value</u>**, which is *<u>the price at which investors are indifferent between buying and selling a security</u>*.

In general, the present value of a cash flow due in *n* years in the future is the amount which, if it were on hand today, would grow to equal the future amount. The **<u>present value</u>** (**<u>PV</u>**) is *<u>the value today of a future cash flow or series of cash flows</u>*.

<u>Finding the present value is</u> called **<u>discounting</u>** *<u>and is simply the opposite of compounding</u>*.

Note the time line:

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0         1         2         3         4         5
|---------|---------|---------|---------|---------|
PV = ?                                         1,338.23
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The formula for discounting can be derived from the FV equation.

$$FV_n = PV(1+i)^n$$

Solving for PV can take several forms:

$$PV = \frac{FV_n}{(1+i)^n} = FV_n\left(\frac{1}{1+i}\right)^n$$

Numerical Solution

Divide $1,338.23 by 1.06 five times, or by $(1.06)^5$, to find PV = $1,000. Thus, the present value of $1,000 is equivalent to the future value of $1,338.23 discounted for five years at 6% interest.

Financial Calculator Solution

Enter N = 5, I/Y = 6, PMT = 0, and FV = 1,338.23, and then press CPT PV to get PV = -1,000. Again, note that the negative sign simply indicates the amount is an investment.